
Delta Hedging

Theory & Application



Background on Fundamentals

What is Delta?

- Before considering *Delta Hedging*, let's first review Delta
- Definition: In options trading, Delta is the measure of how the value of an option changes with respect to changes in the value of the underlying contract
- Typically noted by the Greek letter: Δ



What is Delta?

- Delta (in absolute value, ignoring negative sign) can also be taken as an approximation of the probability that the option will finish in-the-money



What is Delta? (cont'd)

- Delta of a *call* option must be between 0 and 1
 - Since the value of a call option does not change more quickly than the value of the underlying, the maximum value for delta is 1
 - A call option does not move in the opposite direction of the underlying, hence the delta cannot be lower than zero (negative)
- For example, a call option with a delta of 1 (or 100 in trading jargon) will increase or decrease in value by the same amount as the change in the value of the underlying



What is Delta? (cont'd)

- Deep in-the-money calls have deltas at or close to 1
- Far out-of-the-money options have deltas approaching zero
- At-the-money calls have delta values close to 0.5
- *Put* options always have negative deltas, between 0 and -1
 - Because values of puts move in the opposite direction of changes in the price of the underlying
- An underlying contract always has a delta of 1



Terminology

- In trading jargon, option traders often drop the decimal point in describing deltas
 - So a delta of 0.62 may be called 62
- This convention is based on the fact that portions of futures contracts can not be transacted
- Originally adopted in equity markets where a '1 lot contract' is 100 shares of a stock



Delta Neutral

- ‘Neutral’ in that the total delta of the portfolio transactions is zero
 - Total delta position of the portfolio is the aggregate (arithmetic sum) of the transaction deltas
 - Positions with offsetting positive and negative deltas
- And neutral in that the portfolio position is unbiased in terms of *direction* of any price movement of the underlying contract



Delta Neutral (cont'd)

- If a delta neutral position is maintained (by adjusting hedge ratio), the value arbitrage depends only on the volatility of the underlying, not the futures price direction
 - Betting on volatilities not price



Delta Neutral (cont'd)

- If a portfolio's total delta position is negative, then it indicates a downward bias for the underlying futures contract prices
 - A negative delta can be considered equivalent to being short in the underlying market, hence exposed to the same directional price risk
- Conversely, if a portfolio's total delta position is positive, then it indicates an upward price bias
 - A positive delta can be considered equivalent to being long in the underlying market, hence exposed to the same directional price risk



Delta Neutral (cont'd)

- In order to establish a neutral or unbiased hedge, for options purchased one must also sell an appropriate number of underlying contracts
 - The appropriate number of contracts is determined by the delta or *hedge ratio*
- This is the basis for delta hedging



Delta Neutral (cont'd)

- If the factors affecting the option valuation change, then the delta of the position will change
 - Hence, the delta of an option changes as market conditions change
- In order to compensate for this during the life of the option, adjustments need to be made to the underlying futures positions if the portfolio is to be kept delta neutral



Models and Theoretical Value

Black Option Model

- Developed by Fisher Black in 1976
 - The same ‘Black’ of Black-Scholes fame
- Designed to address valuation for futures traded options
 - B-S was based on stock options
- Closed-form solution



Black-Scholes & Black Models

Assume Frictionless Market

- No transaction costs
- No tax implications
- Financing rate same for borrowing & lending for all traders
 - In practice, can be large spread between borrowing & lending
 - Fortunately, the interest rate element in the models is of lesser importance in the valuation
- Underlying contracts trade freely (e.g. without limits on price moves)



Model Assumptions

- Fixed volatility and interest rate over the life of the option being valued
- Lognormal price distribution for underlying contract
- Known future price volatility of the underlying contract for the life of option
- Known future interest rate for the life of the contract



Model Assumptions

- Realistic assumptions? Generally not
 - Certainly not for knowledge of future volatility and interest rates
 - Fixed volatility and lognormal assumptions often not valid
 - Tax consequences may vary depending on liquidation timing and other trade factors, as well as a trader's unique tax considerations



Model Assumptions

- How big of an issue in valuation accuracy?
 - It depends...
- What if the volatility and interest rate assumptions have changed?
 - Presumed by the models to be fixed over the life of the option for which the theoretical value is being determined
- Trader was providing the input to the model as if somehow the future volatility was known



Hedge Ratio

- Note that *Hedge Ratio* is also at times used as the term to describe risk management elements other than Delta Hedges (e.g. the amount of an exposure that has been hedged)
- The Hedge Ratio (the delta) gives the appropriate ratio of underlying contracts to options required to maintain a neutral hedge



Adjusting the Hedge Ratio

- In order to maintain a position near a 'delta neutral' level
- Delta of zero



Frequency of Adjustment of the Delta Hedge

- Professional trader / dealer advantage
 - Lower transaction cost if can buy at the bid and sell at the ask
 - But still some level of transaction cost for commissions or operating costs
- Degree of precision in infrequent hedge ratio adjustment



Arbitrage

- Fair value (theoretical) vs. Market price
- Multiple bets necessary to reach theoretical fair value



Practical Trading Techniques

- Adjusting the hedge ratio by rebalancing the portfolio at regular intervals
 - And more frequently when market conditions change rapidly
- Using gamma to estimate how delta position is changing
 - As a guide in maintaining delta neutral position



Practical Trading Techniques

- The option transaction and associated delta hedges are typically closed-out by:
 - Selling any options that are in-the-money (or exercising the option into a futures contract and selling the futures)
 - Trading conventions may differ between exchanges
- Any out-of-the money options expiring worthless
- Liquidating any open futures used for the delta hedge



Practical Trading Techniques

- The positions do not necessarily have to be held until maturity (option expiration), but can be closed out any time based on profit situation of the position and new market expectations



Obstacles & Flaws

- Transaction costs of hedge ratio adjustments
- Simplifying assumptions of the options valuation models



Summary

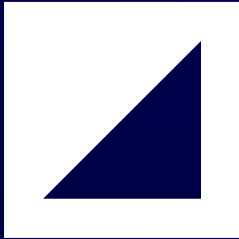
- Delta Hedging is a widely used technique for managing option portfolios
 - Routinely used by professional traders
- Effective use requires knowledge of options fundamentals, pricing, futures/OTC trading practices and quantitative models



Advice & Support

- For advice and implementation support for risk management and trading strategies, RISK Ltd can provide resources as needed for your organization
- Risk Ltd's advisors are experts experienced in both risk management and trading
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